OCR
RECOGNISING ACHIEVEMENT

Friday 13 January 2012 – Morning

A2 GCE MATHEMATICS

4729 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P is projected with speed $40\,\mathrm{m\,s^{-1}}$ at an angle of $35°$ above the horizontal from a point O. For the instant 3 s after projection, calculate the magnitude and direction of the velocity of P. **[5]**

2

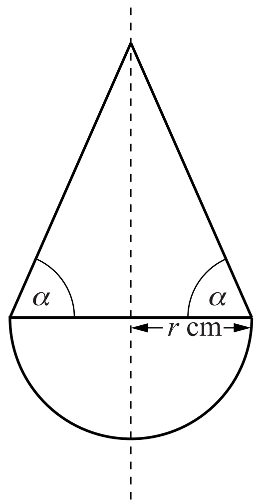


Fig. 1

A child's toy is a uniform solid consisting of a hemisphere of radius r cm joined to a cone of base radius r cm. The curved surface of the cone makes an angle α with its base. The two shapes are joined at the plane faces with their circumferences coinciding (see Fig. 1). The distance of the centre of mass of the toy above the common circular plane face is x cm.

[The volume of a sphere is $\frac{4}{3}\pi r^3$ and the volume of a cone is $\frac{1}{3}\pi r^2 h$.]

(i) Show that $x = \dfrac{r(\tan^2\alpha - 3)}{8 + 4\tan\alpha}$. **[4]**

The toy is placed on a horizontal surface with the hemisphere in contact with the surface. The toy is released from rest from the position in which the common plane circular face is vertical (see Fig. 2).

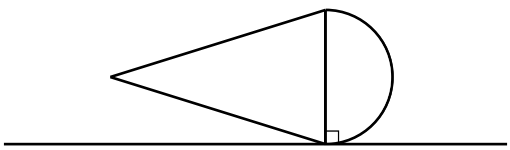
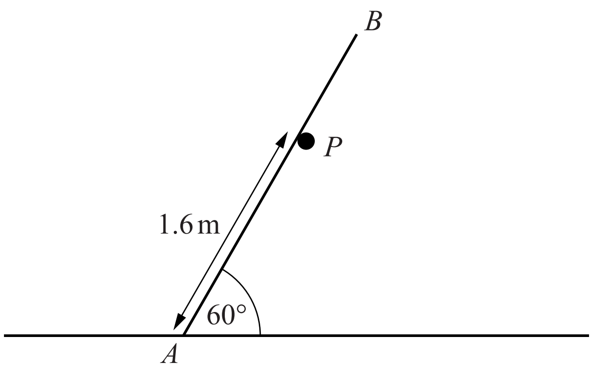


Fig. 2

(ii) Find the set of values of α such that the toy moves to the upright position. **[3]**

3



A uniform rod AB of mass 10 kg and length 2.4 m rests with A on rough horizontal ground. The rod makes an angle of 60° with the horizontal and is supported by a fixed smooth peg P. The distance AP is 1.6 m (see diagram).

(i) Calculate the magnitude of the force exerted by the peg on the rod. **[3]**

(ii) Find the least value of the coefficient of friction between the rod and the ground needed to maintain equilibrium. **[5]**

4 A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is fixed at a point A which is 0.6 m above a smooth horizontal table. P moves on the table in a circular path whose centre O is vertically below A.

(i) Given that the angular speed of P is 2.5 rad s^{-1}, find

(a) the tension in the string, **[4]**

(b) the normal reaction between the particle and the table. **[3]**

(ii) Find the greatest possible speed of P, given that the particle remains in contact with the table. **[5]**

5 A car of mass 1500 kg travels up a line of greatest slope of a straight road inclined at 5° to the horizontal. The power of the car's engine is constant and equal to 25 kW and the resistance to the motion of the car is constant and equal to 750 N. The car passes through point A with speed 10 m s^{-1}.

(i) Find the acceleration of the car at A. **[5]**

The car later passes through a point B with speed 20 m s^{-1}. The car takes 28 s to travel from A to B.

(ii) Find the distance AB. **[7]**

[Questions 6 and 7 are printed overleaf.]

6 A small ball of mass 0.5 kg is held at a height of 3.136 m above a horizontal floor. The ball is released from rest and rebounds from the floor. The coefficient of restitution between the ball and floor is e.

(i) Find in terms of e the speed of the ball immediately after the impact with the floor and the impulse that the floor exerts on the ball. **[4]**

The ball continues to bounce until it eventually comes to rest.

(ii) Show that the time between the first bounce and the second bounce is $1.6e$. **[2]**

(iii) Write down, in terms of e, the time between

(a) the second bounce and the third bounce,

(b) the third bounce and the fourth bounce. **[2]**

(iv) Given that the time from the ball being released until it comes to rest is 5 s, find the value of e. **[5]**

7 A particle P is projected horizontally with speed $15\,\mathrm{m\,s^{-1}}$ from the top of a vertical cliff. At the same instant a particle Q is projected from the bottom of the cliff, with speed $25\,\mathrm{m\,s^{-1}}$ at an angle of $\theta°$ above the horizontal. P and Q move in the same vertical plane. The height of the cliff is 60 m and the ground at the bottom of the cliff is horizontal.

(i) Given that the particles hit the ground simultaneously, find the value of θ and find also the distance between the points of impact with the ground. **[6]**

(ii) Given instead that the particles collide, find the value of θ, and determine whether Q is rising or falling immediately before this collision. **[9]**

Question			Answer	Marks	Guidance
1			$v_x = 40\cos 35$ $v_y = 40\sin 35 - 9.8 \times 3$ $v = \sqrt{32.8^2 + 6.46^2}$ or $\tan\theta = 6.46/32.8$ $v = 33.4$ ms^{-1} $\theta = 11.1°$ below horizontal	B1 B1 M1 A1 A1 [5]	Expect 32.8, need not be evaluated. Expect −6.46, need not be evaluated. Use of Pythagoras or relevant trig on cv(v_x) and cv(v_y) AEF; allow 11.2
2	(i)		$h = r\tan\alpha$ $x(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h) = \frac{1}{3}\pi r^2 h \times \frac{h}{4} - \frac{2}{3}\pi r^3 \times \frac{3}{8}r$ $x = \frac{r(\tan^2\alpha - 3)}{8 + 4\tan\alpha}$	B1 M1 A1 A1 [4]	Seen anywhere and in any form. Table of values idea. **AG** www
2	(ii)		$x < 0$ Solve $\tan^2\alpha - 3 < 0$ $\alpha < 60$	B1 M1 A1 [3]	May be implied. Condone = Condone ≤ throughout. SC Use of = or > throughout. Max B0 M1 A0
3	(i)		 $P \times 1.6 = 10g\cos 60 \times 1.2$ $P = 36.75$ N	M1 A1 A1 [3]	Moments about *A*. Allow 36.8
3	(ii)		 $R + 36.75\sin 30 = 10g$ $F = 36.75\cos 30$ $\mu = 31.8/79.6$ $\mu = 0.4(00)$	M1 A1 FT B1 FT M1 A1 [5]	Attempt at resolving vertically or taking moments. May be implied. $R = 79.6(25)$ Expect 31.8. Or second correct equation involving *F* or *R* or both. For use of (their)$F = \mu$(their)R R not = $10g$ or their P from **(i)**. AWRT www. Allow inequality

Question			Answer	Marks	Guidance
4	(i)	(a)	$\sin\theta = ½$ or $\theta = 30$ $T\cos\theta = 0.2 \times 1.2\cos\theta \times 2.5^2$ $T = 1.5$ N	B1 M1 A1 A1 [4]	θ is angle with horizontal. May have angle with vertical. May be seen later. Attempt at resolving horizontally. cv(r) but not $r = 1.2$ Rounding to 1.5
4	(i)	(b)	 $R + T\sin\theta = 0.2g$ $R = 1.21$ N	M1 A1 FT A1 [3]	Attempt at resolving vertically. FT on cv(T)
4	(ii)		$r = \sqrt{1.2^2 - 0.6^2} = 1.2\cos\theta$ $R = 0$ $T_1\sin\theta = 0.2g$ $T_1\cos\theta = 0.2 \times v^2/_r$ or $0.2 \times r\omega^2$ $v = 4.2$ ms^{-1}	B1 B1 B1 M1 A1 [5]	May been seen in **(i)**, must be used in here. May be implied. Attempt at resolving.
5	(i)		25000/10 1500$g$sin5 2500 – 750 – 1500$g$sin5 = 1500a $a = 0.313$	B1 B1 M1 A1 A1 [5]	 1281.1 Attempt at N2L with 4 terms. cv(1500$g$sin5); cv(2500) not 25000. Allow 0.31
5	(ii)		WD against resistance = 750d WD by engine = 25000×28 (= 700000) Change in PE = 1500g × $d$sin5 Change in KE = ±½ ×1500 × (20^2 – 10^2) 25000×28 = ½ ×1500 ×(20^2 – 10^2) + 750d + 1500g × $d$sin5 $d = 234$	B1 B1 B1 B1 M1 A1 A1 [7]	750h/sin5 1500g × h Use of correct formula for KE. Use conservation of energy, at least 3 used including WD by engine.

Question			Answer	Marks	Guidance
6	(i)		$v^2 = 2 \times 9.8 \times 3.136$ $v = 7.84$ Rebound speed = $7.84e$ $I = \pm 0.5(7.84 + 7.84e) = \pm 3.92(1 + e)$	M1 A1 B1 FT B1 FT [4]	Uses $v^2 = u^2 + 2as$ or energy with $u = 0$. Signs must be consistent. Ignore –ve. AEF seen. FT on cv(v).
6	(ii)		$-7.84e = 7.84e - gt$ $t = 1.6e$ **AG**	M1 A1 [2]	Uses a complete method to find t.
6	(iii)	(a) (b)	$t_2 = 1.6e^2$ $t_3 = 1.6e^3$	B1 B1 [2]	
6	(iv)		Time to first bounce is 0.8 s Identify total time is sum of a GP in e $\frac{1.6e}{1-e} = 4.2$ $e = 0.724$	B1 B1 M1 A1 A1 [5]	 Indication of the sum of at least to term in e^4 Equate 3.4 or 4.2 or 5 or 5.8 with attempt at use of formula for sum to infinity of a GP. Allow 21/29
7	(i)		*For P* $4.9t^2 = 60$ $t = 3.5(0)$ *For Q* $0 = 25\sin\theta \times t - ½ \times 9.8 \times t^2$ $\theta = 43.3$ $PQ = (25\cos\theta - 15) \times t_c$ $= 11.2$	M1 A1 M1 A1 M1 A1 [6]	Signs must be consistent. aef

Question			Answer	Marks	Guidance
7	(ii)		$25\cos\theta(t) = 15(t)$ and solving for θ	M1	Equating horizontal components of velocity (or displacement) and solving for θ.
			$\theta = 53.1$	A1	
			For Q $s_{y1} = 25\sin\theta \times t - ½ \times 9.8 \times t^2$	B1	
			For P $s_{y2} = \pm ½ \times 9.8 \times t^2$	B1	
			Using $s_{y1} + s_{y2} = 60$	*M1	
			Solving for t	M1dep*	
			$t = 3$	A1	
			$v = 25\sin\theta - 9.8 \times 3$	M1	Other methods include finding time to max height for Q.
			$v = -9.4$ therefore falling.	A1	
				[9]	
OR	(ii)		$25\cos\theta(t) = 15(t)$ and solving for θ	M1	Equating horizontal components of velocity (or displacement) and solving for θ.
			$\theta = 53.1$	A1	
			For Q $y = x\tan\theta - \frac{gx^2}{2\times(25)^2\cos^2\theta}$	B1	
			For P $y = (60-)\frac{gx^2}{2\times(15)^2}$	B1	
			Equate y and solve for x	*M1	Must include 60.
			Use $x = u\cos\theta t$ to find t	M1dep*	
			$t = 3$	A1	
			$v = 25\sin\theta - 9.8 \times 3$	M1	Other methods include finding time to max height for Q.
			$v = -9.4$ therefore falling.	A1	
				[9]	